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Unmanned Aerial Vehicle (UAV) Control in the Conditions of Limited Perturbation Effect and Incomplete Information on a State

Dissertation summary

for the purpose of obtaining academic degree

Doctor of Philosophy in Engineering

Academic supervisor:

Doctor of Technical Sciences,
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Work General Characteristics

Research Topic Relevance

Various unmanned aerial vehicles are developed and used actively in the modern world. Helicopters with several groups -copters – have gained popularity. They are applied for exploring, photogrammetry, freight transportation. The development of various algorithms for such drone control are pursued [1].

Mracek C., Cloutier J. [2] and Pearson J.D. [3] in their works were one of the first to represent nonlinear dynamic systems in pseudolinear form with matrix elements depending on system state - State Dependent Coefficients (SDC).

Quadrate cost function is applied rather often after the system representation in the proposed by the authors form for control synthesis. This, rather spread practice, nevertheless, leads to the use of Riccati equation solution with parameters depending on a state in the control algorithm - State Dependent Riccati Equation (SDRE) [4].

The finding of Riccati equation solutions is itself a difficult task at the expense of limitations on onboard microcontroller performance.

The given work proposes control algorithm using SDRE method and capable to be fulfilled on relatively low-performance microcontrollers in quadcopter stabilization task solution.

Quadcopter characteristics can change in a flight, for example, it can drop a load or be damaged. These changes may negatively rebound upon the effectiveness of stabilization system work. The given dissertation research offers algorithm that assesses the change in drone characteristics and tries to fend them off.

Besides afore-described problems, the effects can arise related to controlling impact delay and to obtaining the data on a state which strongly affect flight quality at drone flight [5]. The refuse from delay effect record at regulator synthesis can lead to insufficient effectiveness of control algorithm work [6].

Besides afore-described tasks, the one of quadcopter collective interactions represents an interest [7]. The task of enemy UAV collective interception is considered in the dissertation research. The suppositions on stabilizing regulator synthesis in the conditions of action in the system of delay and after-effect impacts are given.

Research Subject – quadcopter control system, which functions in the conditions of incomplete information on current state, drone characteristics and its interaction with an environment.

Research Purpose - the development of design methods for stabilizing nonlinear regulators and observers, acting in the conditions of various interval uncertainties of system parameters, differing from known ones, in the task of drone control.

Research Tasks:

1. To develop methods and algorithms, that’re comfortable for realization on microcontrollers, for nonlinear regulator synthesis, that're also based on Riccati equation with the parameters depending on a state;
2. To develop object state observer that effectively acts in the interval uncertainty conditions of various nature;
3. To investigate delay effect impact and consequences on a system and to work out algorithm of robust stabilization, allowing to deal with given effects;

4. To compare the obtained algorithm for filtration with known algorithms;

5. To compare known linear regulators with obtained algorithms;

6. To investigate the problem of control of several agents in pursuit task.

Scientific Novelty and Significance. 1) Design and realization algorithms for nonlinear stabilizing regulators for nonlinear dynamic systems describing quadcopter dynamics are developed; 2) Algorithm for the synthesis of adaptive filter, allowing to pursue identification of nonlinear dynamic system parameters, is carried out; 3) Robust regulator synthesis algorithm has been obtained allowing to stabilize system that's under influence of impacts of after-effect and delay in control; 4) Control task for several drones is considered as the problem of optimal control, i.e. differential game with null sum.

Work Practical Significance. The results can be used at solving stabilization and filtration tasks on quadcopter control onboard systems.

Reliability of Results is confirmed by strict mathematical conclusions and numeric mathematical modelling.

Author's Own Contribution is in the development of methods and algorithms of control law design, of methods and algorithms of the assessment of being observed nonlinear system as well as in the pursuing of numeric experiments.

Research Methods include proved methods for dynamic system research and the synthesis of regulators. There are used, in particular: methods of stability and control theories as well as analytical design methods for optimal systems. The check of obtained differential equations was held in software package Maple. Computer modelling was pursued in MATLAB Simulink package.

Work Approbation. Dissertation research results were reported on the following conferences:

1. Semion A.A. Nonlinear Adaptive Filter and Control of Quadcopter // Moscow Workshop on Electronic and Network Technologies (MWENT-22), Moscow 9-11 June 2022;
2. Semion A.A. Presnova A.P. Optimal Control of Car Active Suspension Control under Delays // XVI International Conference "Stability and Oscillations of Nonlinear Control Systems" (Pyatnitskiy's Conference) June 1 - 3, 2022, ICS RAS, Moscow, Russia;
3. Semion A.A. Adaptive coordinate control of nonlinear uncertain object // 13th All-Russia control conference (VSPU-2019) (Moscow), 17-20 June 2019;
4. Semion A.A. A method for realization of nonlinear state-dependent coefficients regulators based on microcontroller memory // Moscow Workshop on Electronic and Network Technologies Together with Siberian Conference on Control and Communication (Moscow) 14-16 March 2018;
5. Semion A.A. Inverted pendulum control with state-dependent coefficients regulator // Intercollegiate Scientific-Technical Conference of Students, PhD Students and Young Specialists named after Armenskiy E.V. (Moscow) 19 February – 01 March 2018;
6. Semion A.A. Estimation of memory usage in implementation of nonlinear regulators with state-dependent coefficients in quasilinear control systems // International Scientific Conference of Students, PhD Students and Young Scientists “Lomonosov-2017” (Moscow) 10-14 April 2017.

Work Short

Introduction narrates the analysis of work relevance and novelty; the work abstract is presented.

First Chapter formulates quadcopter stabilization task.

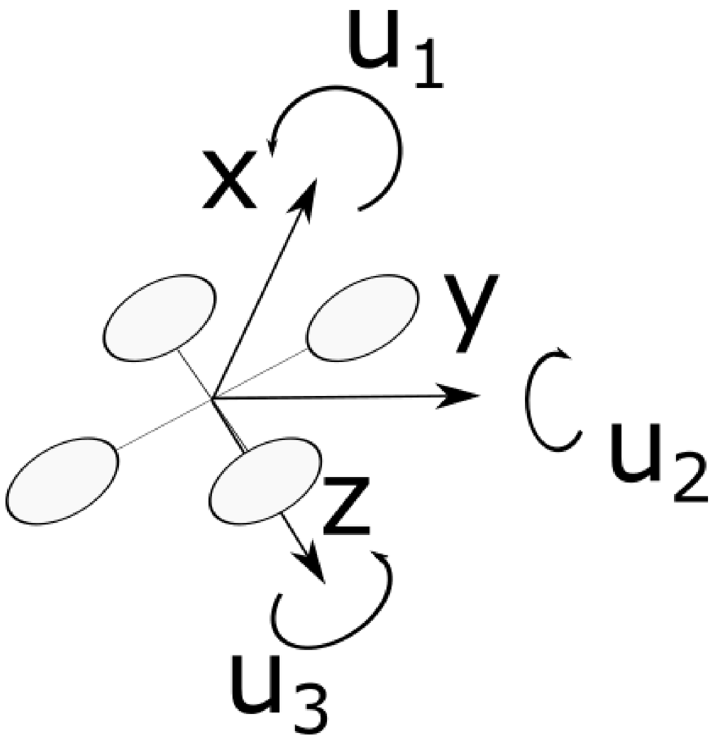

Figure 1. Quadcopter body frame

Nonlinear dynamic system is given that describes quadcopter rotation in a space in quaternion algebra.

Quadcopter dynamics can be described by the following nonlinear systems:

$$\begin{cases} \frac{d}{dt}p(t) = \frac{1}{I_x}\left[u_1(t) - \left(I_z - I_y\right)q(t)r(t)\right] \\ \frac{d}{dt}q(t) = \frac{1}{I_y}\left[u_2(t) - (I_x - I_z)p(t)r(t)\right] \\ \frac{d}{dt}r(t) = \frac{1}{I_z}\left[u_3(t) - \left(I_x - I_y\right)p(t)q(t)\right] \\ \frac{d}{dt}\lambda_0(t) = \frac{1}{2}[-p(t)\lambda_1(t) - q(t)\lambda_2(t) - r(t)\lambda_3(t)] \\ \frac{d}{dt}\lambda_1(t) = \frac{1}{2}[p(t)\lambda_0(t) + r(t)\lambda_2(t) - q(t)\lambda_3(t)] \\ \frac{d}{dt}\lambda_2(t) = \frac{1}{2}[q(t)\lambda_0(t) - r(t)\lambda_1(t) + p(t)\lambda_3(t)] \\ \frac{d}{dt}\lambda_3(t) = \frac{1}{2}[r(t)\lambda_0(t) + q(t)\lambda_1(t) - p(t)\lambda_2(t)] \end{cases} \quad (1)$$

where $p(t)$, $q(t)$, r(t) – angular velocities in quadcopter body frame, $\lambda_0(t), \lambda_1(t), \lambda_2(t), \lambda_3(t)$ – components of quaternion describing quadcopter rotation in a space. $u_1(t)$, $u_2(t)$, $u_3(t)$ – torques, being created by motor rotation difference along x, y, z axes, respectively.

Second Chapter describes the algorithm of stabilizing regulator with parameters depending on a state.

Being considered nonlinear, controlled and being observed system:

$$\begin{aligned} &\frac{d}{dt}x(t) = f\big(x(t)\big) + D\big(x(t)\big)w(t) + B\big(x(t)\big)u(t) \quad , \\ &x(t_0) = x_0 \\ &y(t) = Cx(t), \\ &u(t) \in U, w(t) \in W, t \in \left[t_0, t_f\right], \end{aligned} \tag{2}$$

where $x(\cdot) \in C^1\big(\left[t_0, t_f\right], R^n\big)$,$u(\cdot) \in C^1\big(\left[t_0, t_f\right], R^r\big)$,$w(\cdot) \in C^1\big(\left[t_0, t_f\right], R^k\big)$.

Here $x(t)$ – state of system $x \in \Omega_x$; $x_0 \in \Omega_x$ – system initial state; $y \in R^m$, $m \leq n$ – system output; $u(t)$- control; $w(t)$– perturbation.

We shall consider perturbation $w(t)$ as an effect of some player-enemy hindering control task successful implementation. Various perturbations can come out as, for instance, various airflows.

Let introduce cost function for a differential game:

$$J(x, u, w) = \frac{1}{2} \lim_{t_f \to \infty} \int_{t_0}^{t_f} \{y^{\mathrm{T}}(t)Qy(t) + u^{\mathrm{T}}(t)Ru(t) - w^{\mathrm{T}}(t)P(t)w(t)\}dt \tag{3}$$

Let functions and matrices $f\big(x(t)\big)$ and $\partial f\big(x(t)\big)/\partial x_i$, $D\big(x(t)\big)$, $B\big(x(t)\big)$ and $\partial D\big(x(t)\big)/\partial x_i$, $\partial B\big(x(t)\big)/\partial x_i$, $i = 1, \cdots, n$ be continuous along $x \in \Omega_x$ and $f(0) = 0$, moreover, $\mathrm{D}\big(x(t)\big) \neq 0$, $\mathrm{B}\big(x(t)\big) \neq 0$, $x(t) \in \Omega_{\mathrm{x}}$.

At the fulfillment of given suppositions and SDC linearization use, initial nonlinear system (2) can be represented in the form of the model:

$$\frac{d}{dt}x(t) = A\big(x(t)\big)x(t) + D\big(x(t)\big)w(t) + B\big(x(t)\big)u(t),$$
$$x(t) = x_0,\ y(t) = Cx(t), \tag{4}$$

where control laws $u(t)$ and $w(t)$ are defined in the following way:

$$u(t) = -R^{-1}B^{\mathrm{T}}\big(x(t)\big)S\big(x(t)\big)x(t),$$
$$w(t) = P^{-1}D^{\mathrm{T}}\big(x(t)\big)S\big(x(t)\big)x(t). \tag{5}$$

Positively defined matrix $S\big(x(t)\big)$ is the solution of matrix equation of Riccati type with parameters depending on a state

$$S\big(x(t)\big)A\big(x(t)\big) + A^{\mathrm{T}}\big(x(t)\big)S\big(x(t)\big) - S\big(x(t)\big)\big[B\big(x(t)\big)R^{-1}B^{\mathrm{T}}\big(x(t)\big) - D\big(x(t)\big)P^{-1}D^{\mathrm{T}}\big(x(t)\big)\big]S\big(x(t)\big) + C^{\mathrm{T}}QC = 0 \tag{6}$$

The given control algorithm assumes that quadcopter onboard computer is capable to find equation (6) solution with sufficient speed and online. The algorithms of Riccati equation solution search may be effective insufficiently because of the narrowness of calculation resources of quadcopter onboard electronics at the moment of work introduction. It’s proposed to calculate preliminary with predetermined accuracy all necessary solutions of equation (6), dividing the space of states into a net and calculating gain coefficients in this net crossings.

Schematic representation for state space elements which’re responsible for quaternion vector part is given on Figure 2.

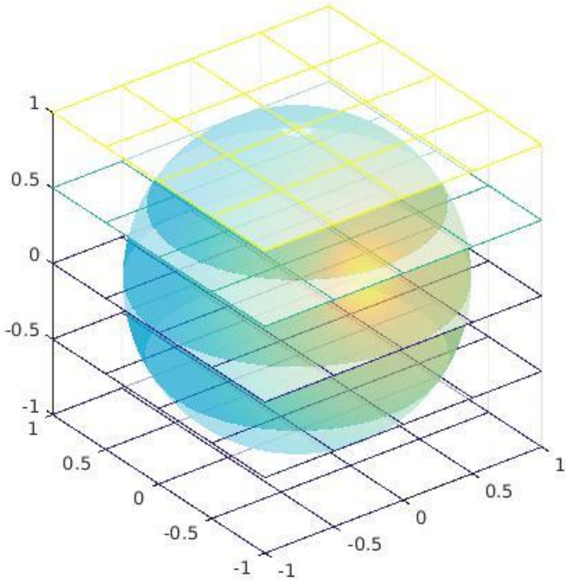

Figure 2. Net in a sphere

Dissertation research put forward proposal on organizing the storage of coefficients in the memory of controlling device as massive of structures which include the value of gain coefficient ($R^{-1}B^{\mathrm{T}}\big(x(t)\big)S\big(x(t)\big)$). It's proposed additionally to record memory addresses on memory elements with gain coefficients for coordinates being nearby in state space.

The calculations of memory necessary size are given for algorithm realization for various accuracies of UAV rotation representation.

Chapter three considers the design of algorithm for adaptive observer and the control which uses adaptation results. Copter's parameters can change during flight, for instance, as a result of load drop or damage.

Let the parameters of matrix $A\big(x(t)\big)$ in (4) are not exactly known but just intervals are known wherein these parameters are. Let suppose that it's possible to divide $A\big(x(t)\big) = A_{и}\big(x(t)\big) + \mathrm{A}_{\alpha}(t)$, where $A_{и}\big(x(t)\big)$ – for certain known part of matrix $A\big(x(t)\big)$, and $\mathrm{A}_{\alpha}(t)$ – the part of matrix $A\big(x(t)\big)$ that is not known exactly. Then system (4) is being represented in the form of:

$$\begin{cases} \dfrac{d}{dt}x(t) = \left[A_и\big(x(t)\big) + \mathrm{A}_\alpha(t)\right]x(t) + D\omega(t) + Bu(t), \\ y = Cx(t) + \eta(t), \\ x(t_0) = x_0. \end{cases} \tag{7}$$

It's needed to build up the evaluation of useful process $\hat{x}(t)$ on the background of noises, according to observations $y(t)$, and that's the best in terms of cost function:

$$J_1(\varepsilon) = M\left[\Psi\big(\varepsilon(t)\big)\right] = M\left[\frac{1}{2}\varepsilon^{\mathrm{T}}\big(t_f\big)F\varepsilon\big(t_f\big) + \frac{1}{2}\int_{t_0}^{t_f}\{\varepsilon^{\mathrm{T}}(t)Q\varepsilon(t)\}dt\right]. \tag{8}$$

Let represent filter structure in the form:

$$\begin{cases} \dfrac{d}{dt}\hat{x}(t) = \left[A_и\big(\hat{x}(t)\big) + A_н(t)\right]\hat{x}(t) + Bu(t) + L(t)[y(t) - C\hat{x}(t)] \\ \hat{x}(t_0) = \bar{x}_0, \\ A_н(t_0) = A_н^0 = \mathrm{A}_\alpha(t_0) \end{cases} \tag{9}$$

Here $A_н(t)$ – additive, adjusted by optimization algorithm, $L(t) = P(t)CN^{-1}(t)$, where matrix $P(t) = M[\varepsilon(t)\varepsilon^{\mathrm{T}}(t)]$ is the solution of Riccati differential equation

$$\frac{d}{dt}P(t) = \left[A_и\big(\hat{x}(t)\big) + A_н(t)\right]P(t) + P(t)\left[A_и\big(\hat{x}(t)\big) + A_н(t)\right]^{\mathrm{T}} -$$
$$P(t)C^{\mathrm{T}}N^{-1}(t)CP(t) + BW(t)B^{\mathrm{T}}, \tag{10}$$

$$P(t_0) = 0.$$

The given equation solution is proposed to be realized online with the rate comparable with the dynamics of an object.

Optimization algorithm design is represented as:

$$\frac{d}{dt}\alpha_н(t) = M\left[\left\{\frac{\partial C\hat{x}(t)}{\partial\alpha_н}\right\}^{\mathrm{T}}\{y(t_1) - C\hat{x}(t_1)\}\right],$$
$$\alpha_н(t_0) = \alpha_0, \tag{11}$$
$$t_1 = t - \gamma, \gamma \neq 0$$

Here $\alpha_н(t)$- the elements of matrix $A_н(t)$.

As a control, it is proposed to use a regulator with discretely changing parameters that is offered in Chapter Second.

Time interval of object functioning can be divided into lapses that are equal by longitude, and at each lapse beginning, the corresponding values of $\hat{x}(t)$ and $A_{и}(\hat{x}) + A_{н}(t)$ can be fixed.

Inside each of the lapse, regulator parameters do not change and are presented as

$$u_{i+1}(t) = K_i\hat{x}(t) = -R^{-1}B^{\mathrm{T}}S_i\hat{x}(t),$$

where S_i – Riccati equation solution of kind

$$S_i[A_{и}(\hat{x}_i) + A_{н}(t_i)] + [A(\hat{x}_i) + A_{н}(t_i)]^{\mathrm{T}}S_i - S_iBR^{-1}B^{\mathrm{T}}S_i + Q = 0, \quad (12)$$

Here $\hat{x}_i$ - $\hat{x}(t)$ value at the moment $t = t_i$, $A_{н}(t_i)$ – $A_{н}(t)$ value at the moment $t = t_i$.

Time lapse length is chosen in a way that onboard computer has to be in time with finding Riccati equation (12) solution.

Thus, the structure of filtration and control system is organized in the form presented on Figure 3.

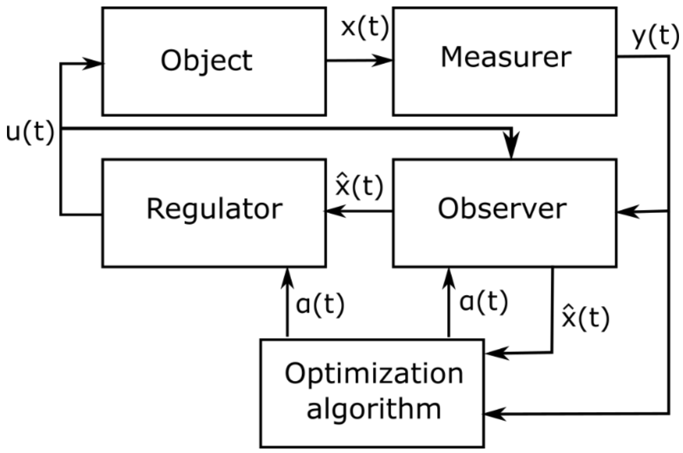

Figure 3 Flowchart of control system structure

Forth Chapter considers system, functioning in the condition of delay and aftereffect impacts. Such impacts can be revealed, for instance, because of delays in circuits of control by copter motors.

There’s considered the system of kind:

$$\frac{d}{dt}x(t) = f(x(t),\tau) + B(x)u(t,\gamma),$$
$$x(t_0) \in X_0, \tag{13}$$

Where $\tau \subset T \subset R^+$ - aftereffect value, $\gamma \subset \Gamma \subset R^+$ - control delay value.

Let suppose that $f_i(x(t),\tau)$, $b_{ij}\big(x(t)\big)$, $i = 1,\dots,n$, $j = 1,\dots,r$ – the elements of matrices $f(x,\tau)$ and $B(x)$, respectively, as well as their derivatives $\frac{\partial f_i(x(t))}{\partial x_k(t)}$, $\frac{\partial f_i(x(t))}{\partial t}$, $\frac{\partial b_{ij}(x(t))}{\partial x_k(t)}$, $\frac{\partial b_{ij}(x(t))}{\partial t}$ are continuous relatively $x(t)$ and t for $i,k = 1,\dots,n, j = 1,\dots,r$. Let represent a control in the form of linear function relatively an object state (13), i.e. $u(t,\gamma) = Kx(t-\gamma)$.

Let delay and aftereffect values be rather small. Then, using the definition of a derivative in null state neighborhood, the system (13) can be presented in the form

$$\frac{d}{dt}x(t) = \left[I + A_\tau\tau + B_\gamma K\right]^{-1}\left[[A_1 + A_\tau + \alpha_1(x(t),\tau)]x(t) + [B_1 + \beta_1(x(t),\gamma)]u(t) + \Im_1(x(t),\alpha_1(x),\beta_1(x),\gamma)\right] \tag{14}$$

Let write down the equation of the first approximation for the system (14).

$$\frac{d}{dt}z(t) = [A + \alpha(x(t),\tau)]z(t) + [B + \beta(x(t),\gamma)]u_z(t),$$
$$z(t_0) = x_0^* \in X_0. \tag{15}$$

Here $[A + \alpha(x(t),\tau)] = \left[I + A_\tau\tau + B_\gamma K\right]^{-1}[A_1 + A_\tau + \alpha_1(x(t),\tau)]$, $[B + \beta(x(t),\gamma)] = \left[I + A_\tau\tau + B_\gamma K\right]^{-1}[B_1 + \beta_1(x(t),\gamma)]$.

Let Ω – set of possible trajectories $\alpha(x(t),\tau)$ and $\beta(x(t),\gamma)$. Let call the worst parameters $\alpha^*,\beta^* \in \partial\Omega$ those ones which at a stable system, no more comes to balance point, and which for, unstable system gets large speed.

The examples of one-dimensional systems having stability (on the left) and not having it are presented on Figure 4. Trajectories, marked with dotted line, correspond to the trajectories of systems with desirable parameters.

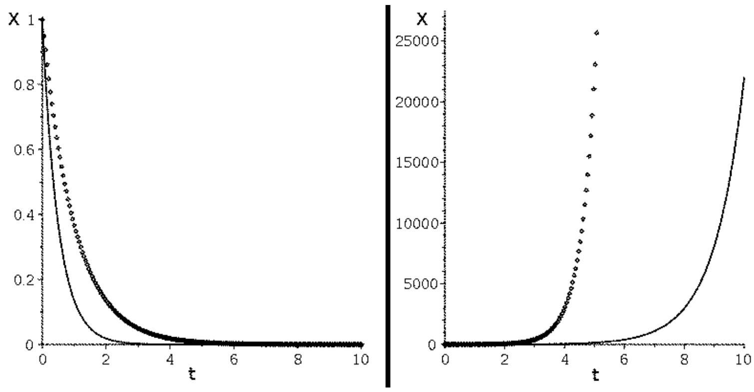

Figure 4. Examples of trajectories of one-dimensional stable and unstable systems

We will implement regulator synthesis, i.e. matrix K search, basing on first approximation (15) model of object (14), the model is of kind

$$\frac{d}{dt} z_M(t) = [A + \alpha^*] z_M(t) + [B + \beta^*] u_M(t) \quad , \tag{16}$$

$z_M(t_0) = x_0$.

Let introduce cost function:

$$J(z_M(t), u_M(t)) = \lim_{t_f \to \infty} \left[\int_0^{t_f} \{ z_M^{\mathrm{T}}(t) Q z_M(t) + u_M^{\mathrm{T}}(t) R u_M(t) \} dt \right] \tag{17}$$

Optimal control for model (16) with cost function (17) will have the following kind [8]:

$$u^*(t) = K z_M(t) = -R^{-1} [B + \beta^*]^{\mathrm{T}} S z_M(t) \tag{18}$$

here, positively defined matrix S is equation Riccati solution

$$S[A + \alpha^*] + [A + \alpha^*]^{\mathrm{T}} S - S[B + \beta^*] R^{-1} [B + \beta^*]^{\mathrm{T}} S + Q = 0, \tag{19}$$

Statement. For finding $\alpha^*, \beta^* \in \Omega$, there can be considered the roots $\lambda_i(\tau)$ of characteristic equation $det(A_z - \lambda \mathrm{I}) = 0$. It's necessary to choose parameter in a way that $Re(\lambda_i(\tau^*)) \geq Re(\lambda_i(\tau))$, $\forall i$, $\forall \tau \neq \tau, \tau \subset T, \tau^* \subset T$.

As an alternative way to find α^*, β^*, it's proposed to consider the norm $M = \|z(t,\tau)\|^2 = \frac{1}{2} z^{\mathrm{T}}(t,\tau) z(t,\tau)$. Desirable trajectories of the system lead to the biggest derivative by time for norm M : $\dot{M}(\tau^*) > \dot{M}(\tau)$, $\forall\tau \neq \tau^*$, $\tau \subset T, \tau^* \subset T$.

For to find α^* to system (16), considered without control, one of the proposed methods is applied. After finding α^*, the initial system is considered as system (16) with control (18), built up at $\beta^* = 0$:

$$u(t) = -R^{-1}B^{\mathrm{T}}S_\alpha z(t), \tag{20}$$

where S_α – Riccati equation solution at $\beta^* = 0$.

The delay by control γ has not participated in control synthesis (20) on the given stage.

Let consider the system of the kind

$$\frac{d}{dt} z_M(t) = [A + \alpha(x(t), \tau^*)]z_M(t) + [B_1 + \beta(x, \gamma)]Kz_M(t, \gamma), \tag{21}$$

where $K = -R^{-1}B^{\mathrm{T}}S_\alpha$. Here, matrix S_α – the solution of Riccati equation (19) with the parameter $\tau = \tau^*$, found on the previous stage, and $\beta^* = 0$.

Let find the worst delay value $\gamma^* \subset \Gamma$, applying one of the proposed methods from the previous stage to the system (21).

Aftereffect τ^* and delay γ^* obtained values are applied for the calculation of the solution of Riccati equation $S_{\alpha\beta}$ (19).

The control, that takes into account the worst values of delay and aftereffect, is represented in the form of:

$$u(t) = -R^{-1}[B_1 + \beta(x, \gamma)]^{\mathrm{T}}S_{\alpha\beta}z(t), \tag{22}$$

where S – Riccati equation solution:

$$S_{\alpha\beta}[A + \alpha^*] + [A + \alpha^*]^{\mathrm{T}}S_{\alpha\beta} - S_{\alpha\beta}[B + \beta^*]R^{-1}[B + \beta^*]^{\mathrm{T}}S_{\alpha\beta} + Q = 0.$$

Fifth Chapter considers differential game with null sum with the big number of players wherein there are n + 1 players in a whole: n pursuers and one evader. Pursuers try to catch avoiding from them evader. Meanwhile, the suppositions are made that watching amongst any pair pursuer-evader is mutual, and watching amongst two pursuers is not mutual obligatory and, at least, a one pair pursuer-evader exists such that this pair every member watches the other, and every pursuer watches, at least, a one other partner in pursuit. The example of such game is interception of a scout drone, striving to fly up to a secret object.

Let consider the task of evader pursuit on m-dimensional Euclidean space with game finite time: $t \in [t_0, t_f]$.

Let $y(t) = [y_1(t), y_2(t), \dots, y_m(t)]^{\mathrm{T}}$, i.e. $y(t) \in R^m$ – evader position, and $x_j(t) = [x_{j1}(t), x_{j2}(t), \dots, x_{jm}(t)]^{\mathrm{T}}$, i.e. $x_j(t) \in R^m$, the position of j^{th} pursuer.

Let introduce a distance vector ("sensitivity radius") between an evader and j^{th} pursuer: $z_j(t) \in R^m$ $z_j(t) = x_j(t) - y(t), j = 1,2, \dots, n$.

In more compact view, if $x^{\mathrm{T}} = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \dots, x_n^{\mathrm{T}}]$ and $z^{\mathrm{T}} = [z_1^{\mathrm{T}}, z_2^{\mathrm{T}}, \dots, z_n^{\mathrm{T}}]$, then

$$z(t) = x(t) - \mathbf{1}_n \otimes y(t). \quad (23)$$

Here and then symbol $\otimes$ means product of Kronecker, and $\mathbf{1}_n$- vector-column with singular element of size $n \times 1$.

To estimate actions of pursuers and an evader, dodging a meeting with pursuers, we introduce common cost function, which pursuers strive to minimize and dogging evader - to maximize in the task with null sum.

$$J_{\Sigma}\left(z(\cdot),u_p(\cdot),u_e(\cdot)\right)=J_p\left(z(\cdot),u_p(\cdot)\right)-J_e\left(z(\cdot),u_e(\cdot)\right)$$
$$=\frac{1}{2}z^{\mathrm{T}}(t_f)Fz(t_f) \tag{24}$$
$$+\frac{1}{2}\int_{t_0}^{t_f}\left\{z^{\mathrm{T}}(t)Qz(t)+u_p^{\mathrm{T}}(t)Ru_p(t)-\left(\mathbf{1}_n\otimes u_e(t)\right)^{\mathrm{T}}P\left(\mathbf{1}_n\otimes u_e(t)\right)\right.$$

The result for classical differential game with several pursuers and linear feedback is presented with the following theorem which argumentum for is given in the dissertation text.

Theorem. *Differential game with n-pursuers and a one evader, dodging pursuit with dynamics (23) and cost function (24), is given. The game has solution at condition* $r_p < nr_e$ *if players' strategies are defined by the following expressions:*

$$u_p^0(t)=-\frac{1}{r_p}K(t)z(t),$$
$$u_e^0(t)=-\frac{1}{nr_e}(\mathbf{1}_n^{\mathrm{T}}\otimes I_m)K(t)z(t), \tag{25}$$

where

$$\frac{d}{dt}K(t)=-K(t)\left[-\frac{1}{r_p}I_n+\frac{1}{nr_e}(\mathbf{1}_n\otimes\mathbf{1}_n^{\mathrm{T}}\otimes I_m)\right]K(t)-$$
$$[q_p+q_e]I_n, \tag{26}$$
$$K(t_f)=[k_{pf}+k_{ef}]I_n.$$

Theorem. *Differential game* (23), (24) *has a solution if matrices of cost function* (24) *R and P are connected by relation* $R \prec P$. The theorem proof is presented in the dissertation text.

In the differential game with distributed information, each player adopts a decision on the basis of just such information that's available to him in the given time moment.

Let write down in general view the formula for vector, denoting the distance between j[th] pursuer, evader and the rest of pursuers

$$\tilde{z}_{pj}(t) = x_j(t) - \sum_{i=1}^{n} d_{ij}(t)x_i(t) - f_j(t)y(t). \tag{27}$$

If escaping evader observes other pursuers' actions, he can possess the following information

$$\tilde{z}_e(t) = \sum_{i=1}^{n} e_i(t)x_i(t) - y(t). \tag{28}$$

Strategies for pursuers can look in the following way:

$$u_{pj}^0(t) = -\frac{k_p(t)}{r_p}\tilde{z}_{pj}(t) = -\frac{k_p(t)}{r_p}\left[x_j(t) - \sum_{i=1}^{n} d_{ij}(t)x_i(t) - f_j(t)y(t)\right] \tag{29}$$

for j=1, 2, ..., n.

Let note at the consideration of strategy for escaping evader on that when the escaping one observes several pursuers' actions, it forms a control, trying to dodge "mass center" of all detected pursuers, using available information (28)

$$u_e^0(t) = -\frac{k_e(t)}{r_e}\tilde{z}_e(t) = -\frac{k_e(t)}{r_e}\left[\sum_{i=1}^{n} e_i(t)x_i(t) - y(t)\right]. \tag{30}$$

Parameters $k_p(t)$ and $k_e(t)$ in (29) and (30) are being found from the solutions of the equation

$$\frac{d}{dt}k_p(t) - \left[\frac{r_e - r_p}{r_e r_p}\right]k_p^2(t) - \frac{2}{r_p}k_p(t)k_e(t) + q_p = 0\ , \quad k_p(t_f) = k_{pf}, \tag{31}$$

$$\frac{d}{dt}k_e(t) - \left[\frac{r_e - r_p}{r_e r_p}\right]k_e^2(t) - \frac{2}{r_e}k_p(t)k_e(t) + q_p = 0\ , \quad k_e(t_f) = k_{ef}. \tag{32}$$

Let note at the consideration of strategy for escaping evader (30) on that in the case if the escaping one observes several pursuers in its sensitivity radius, the escaping one would form such control that would try to “fly away” from mass center of all observed pursuers.

In the case when an evader creates artificial noise with the purpose to hinder a pursuer and get advantage in the game, pursuers will receive information on escaping evader with noises.

The Sixth Chapter shows the results of mathematical modelling of the work of the algorithms proposed in previous chapters. Regulator with discretely changing parameters is looked to UAV

$I = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.15 \end{bmatrix}$ at net accuracy $\varepsilon = 0.1$.

As initial conditions, the vector was chosen $(p \quad q \quad r \quad \lambda_0 \quad \lambda_1 \quad \lambda_2 \quad \lambda_3)^{\mathrm{T}} =$
$(0 \quad 0.5 \quad 0.1 \quad 0.95 \quad 0.26 \quad -0.03 \quad -0.14)^{\mathrm{T}}$.

Control is capable to lead a quadcopter in horizontal position and hold it such.

For to demonstrate observer’s work, noisy measurer is introduced, the precise value of one of the inertia tensor components is unknown. UAV control system, consisting of an observer, adaptation algorithm and regulator, leads UAV to horizontal state.

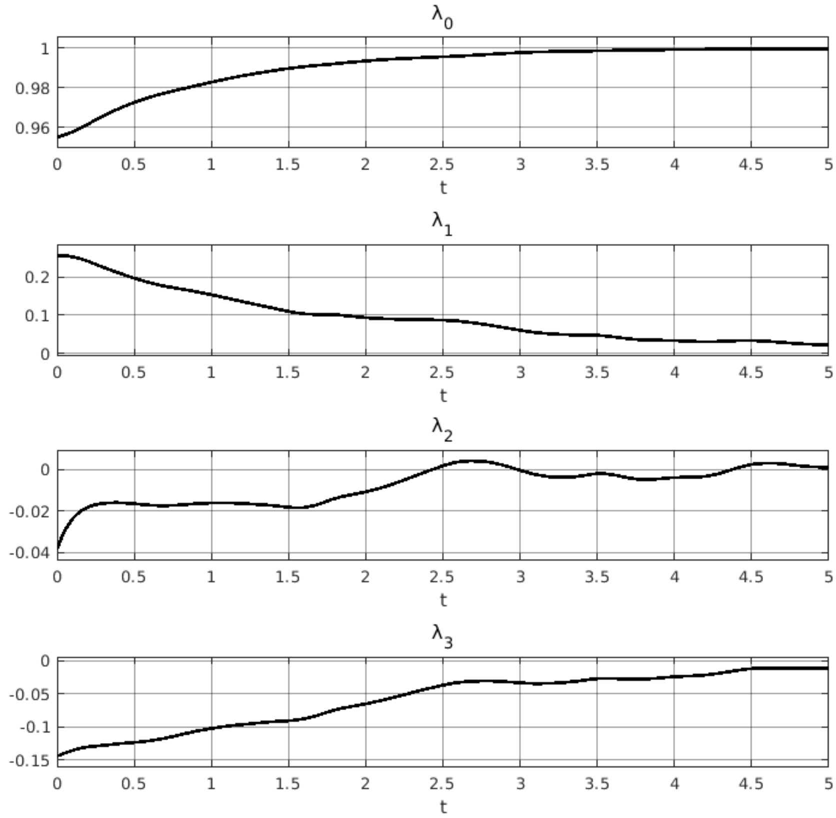

Figure 5. Trajectory of quaternion components' changes

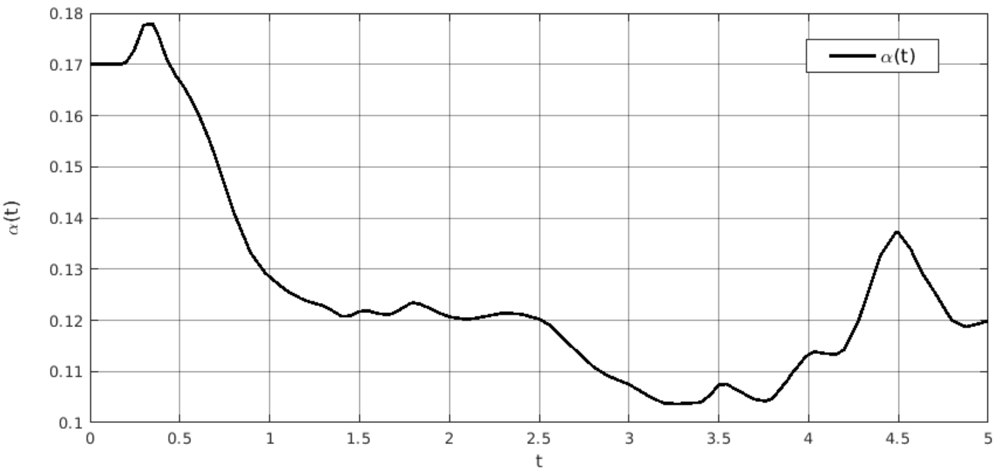

Figure 6. α parameter optimization

Let make comparison of Kalman-Bucy filter efficiency without adaptation and with it. For that, let subtract the value of Kalman-Bucy filter cost function without adaptation from the value of Kalman-Bucy filter cost function with adaptation. Figure 7 demonstrates the graph of this difference. Adaptation absence leads to cost function bigger value because filter doesn't optimize its work in the conditions of uncertainty of parametrical noise.

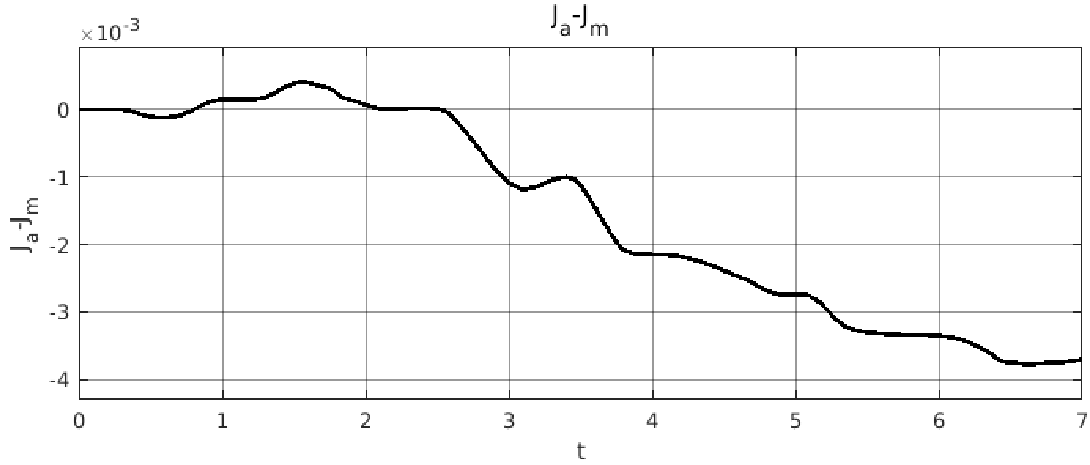

Figure 7. Difference of Kalman-Bucy filter cost function with adaptation and without it

The dissertation Sixth Chapter also demonstrates the work of stabilization algorithm for simple system with delay and aftereffect impact.

Differential game for 3 pursuers and a one evader is modelled. The work presents the graphs of transition processes at classical game, game with distributed information at noise absence and presence.

Work Major Results

1. Algorithm of the realization of SDRE control in the conditions of limitation of hardware resources, which stabilize quadcopter in horizontal position, has been proposed;
2. Kalman-Bucy adaptive filter has been proposed which allows to pursue the identification of quadcopter characteristics and which is capable to counteract noises;
3. SDRE control has been synthetized which uses identification results obtained by filter and stabilizes a drone;
4. Control has been synthetized that’s capable to stabilize system under the influence of delay and aftereffect impacts;
5. Control by the set of pursers and dodging one in the task of differential game with null sum has been synthesized;

6. Numeric modelling of the work of obtained algorithms has been performed in the task of quadcopter stabilization and applications.

Publications on Dissertation Topic

The work major results have been published in the following **articles**:

Works published by the author in peer-reviewed scientific publications included in the international Scopus citation system:

1. Semion A.A. A method for realization of nonlinear state-dependent coefficients regulators based on microcontroller memory // Moscow Workshop on Electronic and Networking Technologies, MWENT 2018 - Proceedings, 14-16 March 2018, Moscow
2. Semion A., Presnova A. Optimal Control of Car Active Suspension Control under Delays // 16th International Conference on Stability and Oscillations of Nonlinear Control Systems (Pyatnitskiy's Conference) (STAB). IEEE, 2022. Ch. 60. pp. 1-4.

Works published by the author in scientific journals included in the list of high-level journals prepared at the Higher School of Economics:

3. Afanasiev V.N., Semion A.A. Differential Games with Several Pursuers and One Evader // Control Problems 2021. №1, pp. 24-35 *(in Russian)*;
4. Semion A.A. Method of Microcontroller Memory Usage at Realization of Nonlinear Regulators with Coefficients, Depending on a State // Journal of Information Technologies and Computing Systems 2017. № 4. pp 64-70 *(in Russian)*;
5. Afanasiev V.N., Semion A.A. Controller with Discrete Variable Parameters // Control Sciences. 2014. № 5. pp 14-19 *(in Russian)*;

Applicant publications in other publications:

6. Afanasiev V.N., Semion A.A. Object control in terms of delay and aftereffect with interval duration // Automation. Modern Technologies, 2020. V. 74, №4 pp. 170-175

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